Progress 01

1)

Let $\underset{\sim}{a} = \underset{\sim}{i} - \underset{\sim}{j}$, $\underset{\sim}{b} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{c} = 4\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$ be vectors; O is the origin and suppose $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$, and $\overrightarrow{OC} = \underset{\sim}{c}$.

a) Find the area of the triangle ABC.

b) Show that the points $O, A, B, and\ C$ are on a plane.

c) Find equation of the plane go through the points $O, A, B, and\ C$.

1) Let A, B, C and D be four distinct points such that $\overrightarrow{OA} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$, $\overrightarrow{OB} = 2\underset{\sim}{i} + \underset{\sim}{j}$, $\overrightarrow{OC} = 4\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\overrightarrow{OD} = 7\underset{\sim}{i} + 4\underset{\sim}{k}$, where O is the origin.

a) Let α be the plane passing through the points A, B and C. Find the equation of α in scalar form.

b) Let l be the straight line passing through A and the midpoint of the line segment BC. Find the equation of l in scalar form.

c) Let β be the plane passing through the point D and is perpendicular to the line l. Find the equation of β in scalar form.

d) Calculate the angle between α and β.

1. (a) Let A, B, C and D be points with coordinates $(4, 5, 1), (0, -1, -1), (3, 9, 4)$ and $(-4, 4, 4)$. Use **vector methods** for the following problems.

   i. Find the vector equation and cartesian equation of the straight line which passes through A and parallel to $\overrightarrow{BC}$.

   ii. Find the area of the triangle BCD.

   iii. Show that A, B, C and D are coplanar.

   iv. Find the vector equation and cartesian equation of the plane $ABCD$.

   (b) Let $\underset{\sim}{a}, \underset{\sim}{b}$ and $\underset{\sim}{c}$ be non zero vectors. Show that,

   i. $[\underset{\sim}{a} + \underset{\sim}{b}, \underset{\sim}{b} + \underset{\sim}{c}, \underset{\sim}{c} + \underset{\sim}{a}] = 2[\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}]$

   ii. $\underset{\sim}{i} \times (\underset{\sim}{a} \times \underset{\sim}{i}) + \underset{\sim}{j} \times (\underset{\sim}{a} \times \underset{\sim}{j}) + \underset{\sim}{k} \times (\underset{\sim}{a} \times \underset{\sim}{k}) = 2\underset{\sim}{a}$